

Absolute Value:

$$|-3| = 3, \text{ and } |3| = 3$$

The absolute value is the symbol $|x|$, this means the output is always positive, as you can see above $|-3|$ becomes 3, and $|3| = 3$, despite of positive number the result remains positive.

$$\text{Assume } |x| = 3$$

X can be either 3 or -3, because if 3 or -3 substituted instead of X, a positive value of 3 will be the result that means two options are available to be the value of $|x| = 3$.

Strategy to solve:

The technique to solve absolute question:

1. Make absolute alone.
2. To eliminate the absolute, make two equations firstly write it as it, secondly change the sign of the answer.
3. Solve each equation to find the value of the variable.

Example: If $|x - 3| = 5$, find the value of x?

Solution:

Following the technique above.

1. The absolute is alone. $|x - 3| = 5$
2. Making two equations:

$$x - 3 = 5$$

$$x - 3 = -5$$

3. Solve each equation:

$$x - 3 = 5, \text{ adding } +3 \text{ to both sides then, } x = 8$$

$$x - 3 = -5, \text{ adding } +3 \text{ to both sides then, } x = -2$$

Answer: $x = 8$ or $x = -2$

Example: if $-2|x - 1| = -8$, then the value of x is ?

Solution:

1. Making absolute alone by dividing both side by -2

$$|x - 1| = 4$$

2. Making two equations:

$$x - 1 = 4$$

$$x - 1 = -4$$

3. Solve per each by adding $+1$ to both sides:

$$x = 5$$

$$x = -3$$

Answer is $x = 5$ or $x = -3$

Example: If $-3 - 2|2 - 2x| = -11$, find the value of x ?

Solution:

1. Make absolute alone by adding $+3$ to both sides $-2|2 - 2x| = -8$, then divide by (-2) for both sides $|2 - 2x| = 4$.

2. Making two equations

$$2 - 2x = 4$$

$$2 - 2x = -4$$

3. Solve per each by -2 on both sides on both equations :

$$-2x = 2$$

$$-2x = -6$$

Divide by -2 for both equations:

$$x = -1$$

$$x = 3$$

Answer $x = -1$ or $x = 3$

Example (SAT): If $|x - 2| = 9$, then what is the value of $|x + 3|$?

- (A) 9
- (B) 11
- (C) -7
- (D) -14
- (E) 4

Solution:

From $|x - 2| = 9$, find x

$$x - 2 = 9 \text{ or } x - 2 = -9$$

Adding +2 on both sides

$$x = 11 \text{ or } x = -7$$

Now the question is looking for $|x + 3|$, to find it substitute the both values of x

When $x = 11$, then $|x + 3| = |11 + 3| = 14$

$x = -7$, then $|-7 + 3| = 4$

Answer is E $|x + 3| = 4$

Absolute Inequality:

The inequality absolute has the same procedure as equal.

Strategy to solve:

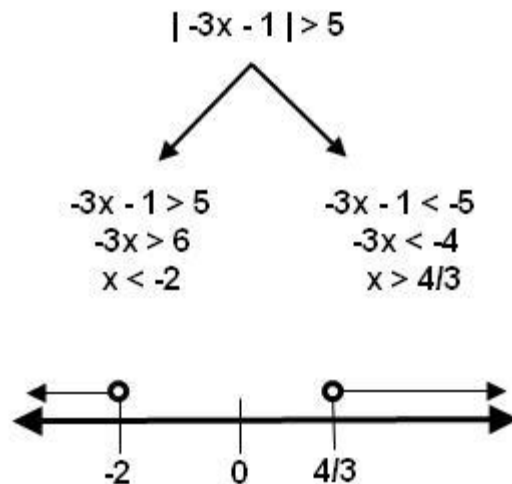
The technique to solve inequality absolute question:

1. Make absolute alone.
2. To eliminate the absolute, make two equations firstly write it as it, secondly flip the inequality sign and change the sign of the answer.
3. Solve each equation to find the value of the variable.

Example: If $|-3x - 1| > 5$, what is the value of x ?

Solution:

Following the procedure:



Answer:

$$x < -2 \text{ or } x > 4/3$$
$$(-\infty, -2) \text{ or } \left(\frac{4}{3}, \infty\right)$$

Example: If $|x + 2| \leq 3$, Find x ?

$$\begin{array}{ccc}
 |x + 2| \leq 3 & & \\
 \swarrow & & \searrow \\
 x + 2 \leq 3 & & x + 2 \geq -3 \\
 x \leq 1 & & x \geq -5
 \end{array}$$

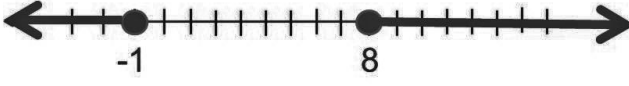
Answer:

$$X: [-5, 1]$$

$$-5 \leq x \leq 1$$

Example: If $|-2x + 7| + 5 \geq 14$, what is the value of x ?

Solution:

$$\begin{array}{c}
 |-2x + 7| + 5 \geq 14 \\
 \quad \quad \quad -5 \quad -5 \\
 |-2x + 7| \geq 9 \\
 -2x + 7 \geq 9 \quad \text{or} \quad -2x + 7 \leq -9 \\
 \quad \quad \quad -7 \quad -7 \quad \quad \quad -7 \quad -7 \\
 \frac{-2x}{-2} \geq \frac{2}{-2} \quad \text{or} \quad \frac{-2x}{-2} \leq \frac{-16}{-2} \\
 x \leq -1 \quad \text{OR} \quad x \geq 8
 \end{array}$$


$$(-\infty, -1] \text{ or } [8, \infty)$$